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Waves in gas centrifuges: a review

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Abstract. Waves in gas centrifuges are generated by scoops for withdrawal of the gas. The physics of the waves and their role in the gas dynamics are under discussion. Strong centrifugal and Coriolis forces have dramatic impact on the properties and dispersion relation of the waves. The conventional sound, vortex and entropy waves split into 3 families with different dispersion. The entropy wave has zero velocity of propagation but variation of temperature in this wave is accompanied by toroidal motion. Pressure is not perturbed. The rest two families of the waves have nonzero velocities of propagation. Upper family has frequency above doubled frequency of rotation of the rotor with exceptional case of the wave (named acoustic wave) propagating exactly in the axial direction. This wave propagates with the conventional sound velocity and is polarized only in the axial direction. Unique property of this wave is the weakest damping due to the molecular viscosity and heat conductivity. All other waves are damped on distances compared with their wavelength. At the conventional parameters of the IGUASU centrifuge the acoustic waves are damped predominantly due to the viscous interaction and heat exchange with the wall of the rotor. This wave is able to propagate on the distance of ≤ 1 m. Numerical experiments show that the waves can affect the axial circulation and gas content in the centrifuge and produce phenomena of resonances. Possible impact of the waves on the process of separation of the isotopes is also under discussion.

1. Introduction

Gas centrifuges (hereafter GC) are explored for uranium enrichment starting with 50th years of the last century. Nevertheless, the physics of the gas flow in the GC is still not fully understood. Especially this is valid in relation to the impact of waves excited by scoops used for withdrawal of the gas from the GC. They produce waves which propagate along the rotor. The properties of the waves in ideal gas compressed in the centrifugal field of the order 10^6 g (g – acceleration of gravity at the Earth surface) are rather specific. The physics of the waves and their impact on the gas dynamics and separation process is still poorly investigated.

Waves in the rotating liquids were studied in many works [1, 2, 3]. The properties of the waves in the gases were studied only in few works and the investigations have been limited by rather moderate rotational velocities [4]. The rotational velocities of the conventional GC is about 600 – 700 m/s at the rotor radius about 6 – 9 cm [5]. Velocities of the rotor above 1000 m/s recently started to discuss [6]. The centrifugal acceleration is about 10^5 – 10^6 g at these parameters. Therefore, pressure of the working gas changes on 3-4 orders of magnitude at 1 cm radius variation. In these conditions conventional acoustic waves are strongly modified. In this review we outline the progress achieved in National Research Nuclear University (MEPHI) in the physics of waves in strong centrifugal field.



2. Physics of waves in strong centrifugal field

2.1. Linear waves in a rotating ideal gas

Let us firstly consider the properties of the waves neglecting dissipative processes: viscosity and thermal conductivity of the gas. For the gas in normal conditions damping of the waves due to dissipative processes on the wave length is weak. Therefore this approach looks reasonable. Gas is compressed to the wall of the rotor in the GC. Its density reduces exponentially to the axis of rotation. Therefore a region near the axis of rotation always exist in the conventional centrifuges where the dissipative processes can not be neglected. The radius of this region can be estimated from comparison of the inertial term in Navier-Stokes equation equal $\rho \frac{\partial v}{\partial t} \approx \rho v \Omega$ with the viscous term $\approx \frac{\eta v}{\lambda^2}$, where λ is the wave length and Ω is the angular frequency of the wave. Taking into account that $\lambda \Omega \approx c$ where c is the sound velocity and that $\eta \approx l \rho c$ we can conclude that dissipative effects can be neglected at radius where $\lambda \gg l$, where l is free path length of molecular. But this is the limit of applicability of hydrodynamical approximation. Therefore in all the region where we can apply hydrodynamical approximation the dissipative processes can be considered as a small perturbation of the solution.

The system of linearized equations for a small amplitude perturbation of the gas in the rotating frame system has a form [7]:

$$\frac{\partial \rho}{\partial t} + \frac{\partial (r \rho_0 v_r)}{r \partial r} + \frac{\partial (\rho_0 v_\phi)}{r \partial \phi} + \frac{\partial (\rho_0 v_z)}{\partial z} = 0, \quad (1)$$

$$\rho_0 \frac{\partial v_r}{\partial t} - \rho \omega^2 r - 2 \rho_0 \omega v_\phi = - \frac{\partial p}{\partial r}, \quad (2)$$

$$\rho_0 \frac{\partial v_\phi}{\partial t} + \rho_0 2 \omega v_r = - \frac{\partial p}{r \partial \phi}, \quad (3)$$

$$\rho_0 \frac{\partial v_z}{\partial t} = - \frac{\partial p}{\partial z}, \quad (4)$$

$$\rho_0 c_p \frac{\partial T}{\partial t} = \frac{\partial p}{\partial t} + v_r \frac{\partial p_0}{\partial r}. \quad (5)$$

where c_p – is the specific heat capacity for constant pressure. p, ρ, T and v_r, v_ϕ, v_z are perturbations pressure, density, temperature and radial, azimuthal and axial components of the velocity.

Parameters of the gas can be presented as the sum of parameters of the rigid body rotation and some deviation

$$v_r = \bar{v}_r, v_z = \bar{v}_z, v_\phi = \bar{v}_\phi, P = p_0 + p, \rho = \rho_0 + \rho, \tau = T_0 + T, \quad (6)$$

where p_0, ρ_0, T_0 – are the rigid body rotation pressure, density and temperature, respectively. $v_r, v_\phi, v_z, p, \rho, T$ – are deviations of radial, azimuthal, axial velocity, pressure, density and temperature from the rigid body rotation values, respectively.

Dependence of the rigid body pressure and density on the radius has a form

$$p_0(r) = p_w \exp\left(\frac{M \omega^2}{2RT_0} (r^2 - a^2)\right), \quad (7)$$

$$\rho_0(r) = \rho_w \exp\left(\frac{M \omega^2}{2RT_0} (r^2 - a^2)\right), \quad (8)$$

where a – is the radius of the rotor, p_w and ρ_w – are the pressure and density of the gas at the wall of the rotor, respectively.

For the ideal gas

$$\rho = \frac{Mp}{RT}. \quad (9)$$

For the deviation from rigid body rotation we have

$$\rho = \frac{Mp}{RT_0} - \frac{\rho_0 T}{T_0}. \quad (10)$$

The system of equations (1-5) has 5 equations. Therefore it has 5 eigenvalues corresponding to 5 types of waves. In the conventional conditions these types of waves consist 2 longitudinal sound waves propagating with the sound velocity in the opposite directions, 2 transversal vortex waves with different polarizations propagating with zero velocity and one scalar entropy wave propagating with zero velocity [7]. In the centrifugal field the number of equations remains the same. Therefore, in the centrifugal field we have the same number of types of waves. But their properties are strongly modified.

Firstly, the relation between perturbation of pressure and density in the wave changes. In the conventional gas they are related by equation $p = \rho c^2$. In the centrifugal field it follows from eqs. (5) and (10) that they are related as

$$\frac{\partial \rho}{\partial t} = \frac{1}{c^2} \frac{\partial p}{\partial t} - \frac{v_r}{T_0 C_p} \frac{\partial p_0}{\partial r}. \quad (11)$$

This relation was found firstly in our work [8]. It has simple explanation. The origin of the last term in eq. (11) is connected with the work of the centrifugal field. Let us consider the process of motion of a small piece of the gas from the initial position 1 in the initial moment 1 to the position 2 separated by the radial distance Δr at the next time moment 2. The conventional adiabatic equation has a form

$$\rho_2 = \rho_1 \left(\frac{p_2}{p_1} \right)^{\frac{1}{\gamma}}, \quad (12)$$

where $\gamma = C_p/C_v$, p_1, ρ_1 are the pressure and density at position 1, p_2, ρ_2 are the pressure and density at position 2, respectively. In the uniform gas p_1 and ρ_1 are constants and we obtain the conventional equation for variation of the pressure and density

$$\delta \rho = \frac{\delta p}{c^2}, \quad (13)$$

where $\delta \rho$ and δp are variations of the density and pressure at the position 2 during time interval Δt .

In the strong centrifugal field the variation of the unperturbed state of the gas with r should be taken into account because entropy of the unperturbed gas varies with r . This gives

$$p_2 = \delta p + p_1 + \frac{\partial p_0}{\partial r} \Delta r, \rho_2 = \delta \rho + \rho_1 + \frac{\partial \rho_0}{\partial r} \Delta r. \quad (14)$$

In this case the connection between variations of the density and pressure takes a form

$$\delta \rho = \frac{\delta p}{c^2} - \frac{\partial \rho_0}{\partial r} \Delta r + \frac{1}{\gamma} \frac{\partial \rho_0}{\partial r} \Delta r. \quad (15)$$

This gives

$$\delta \rho = \frac{\delta p}{c^2} - \frac{(\gamma - 1)}{\gamma} \frac{\rho_0 M \omega^2 r}{RT_0} V_r \Delta t, \quad (16)$$

which is equivalent to eq. (11). In the works by [9] and [10] this effect has been fully neglected. This is one of the reasons why their results can not be applied to the waves in GC.

In work [8] propagation of the linear plane waves in ideal rotating gas has been considered. Two different families of waves with different polarizations and dispersion laws were discovered in the result of solution of the system of equation (1-5) They are defined by the equations:

$$V_r = \frac{Y}{r\rho_0} e^{\frac{M\omega^2 r^2}{4RT_0}}, \quad V_\varphi = \frac{2\omega Y}{i\Omega r\rho_0} e^{\frac{M\omega^2 r^2}{4RT_0}}, \quad V_z = \frac{kp}{\Omega\rho_0}, \quad (17)$$

$$p' = \frac{1}{A} \left(\frac{\omega Y}{c_p T_0} e^{\frac{M\omega^2 r^2}{4RT_0}} - \frac{1}{r} \frac{\partial}{\partial r} \left(Y e^{\frac{M\omega^2 r^2}{4RT_0}} \right) \right), \quad T' = \frac{-i\Omega p' + \rho_0 \omega^2 r V_r}{-i\rho_0 c_p \Omega}, \quad (18)$$

where i – imaginary unit, p' , T' , V_r , V_φ , V_z – amplitude of the perturbation of the pressure, temperature, radial, azimuthal and axial velocities respectively, ρ_0 – rigid body rotation density, Ω – perturbation frequency, k – projection of the wave vector to the axial direction, ω – angular velocity, r – cylindrical radius, c_p – gas specific heat capacity for constant pressure, M – molar mass of the gas, R – gas constant, Y – Whittaker function with arbitrary normalization:

$$Y = WM \left(\frac{B'}{4\sqrt{-A'}}, 0.5, \sqrt{-A'} r^2 \right). \quad (19)$$

Here $A' = -\frac{M^2\omega^4}{4R^2T_0^2} + \frac{k^2\omega^4}{c_p T_0 \Omega^2}$, $B' = -\frac{1}{\Omega^2} \left(k^2 - \frac{\Omega^2}{c^2} \right) (\Omega^2 - 4\omega^2)$, c – sound velocity. The equation $Y(r = a) = 0$, where a – rotor radius, defines us the dispersion relation for the waves in the rotating gas. They split into “upper” and “lower” families of the waves. The “upper” family has degenerate case of the sound wave with dispersion law $\Omega = kc$ and

$$V_r = 0, \quad V_\varphi = 0, \quad V_z = \frac{c p'_w}{\gamma p_w} \exp \left(\frac{(1 - \gamma) \omega^2 (r^2 - a^2)}{2c^2} \right), \quad (20)$$

$$p' = p'_w \exp \left(\frac{\omega (r^2 - a^2)}{2c^2} \right), \quad T' = \frac{p'}{\rho_0 c_p}, \quad \rho' = \frac{p'}{c^2}. \quad (21)$$

Here p_w and p'_w pressure at the rotor wall and its perturbation respectively. This case corresponds to the zero level of excitation of the gas in the radial direction. This wave propagates exactly along the rotational axis and is longitudinally polarised.

The “upper” family of the waves with $\Omega \geq kc$ reduces to the conventional acoustic waves in the limit of quiescent gas. The “lower” family of the waves with $\Omega < kc$ reduces to the so-called “vortex” waves which have zero frequency and velocity of propagation in the quiescent gas [7] (see task on pp. 315-316). These waves have finite velocity of propagation in the rotating gas.

The last type of waves is entropy waves. The centrifugal field also modifies their properties. Let us consider perturbations with $\Omega = 0$. In this case equations (1-5) become

$$\frac{\partial (r\rho_0 V_r)}{r\partial r} + ik\rho_0 V_z = 0, \quad (22)$$

$$-2\rho_0 \omega V_\varphi - \rho' \omega^2 r = -\frac{\partial p'}{\partial r}, \quad (23)$$

$$2\rho_0 \omega V_r = 0, \quad (24)$$

$$kp' = 0, \quad (25)$$

$$\rho_0 \omega^2 r V_r = 0. \quad (26)$$

Equations (22, 24-26) give $V_r = 0$, $p' = 0$ and $V_z = 0$. From the equation of state

$$\rho_0 = \frac{M}{RT_0} p_0, \rho' = \frac{M}{RT_0} p' - \rho_0 \frac{T'}{T_0}, \quad (27)$$

we obtain

$$\rho' = -\rho_0 \frac{T'}{T_0}, \quad (28)$$

where T_0 is the temperature of the unperturbed gas. Substitution of $p' = 0$ into equation (23) gives

$$-2\rho_0\omega V_\varphi - \rho'\omega^2 r = 0. \quad (29)$$

Substitution of eq. (28) into eq. (29) gives the final relationship between gas characteristics in the entropy wave:

$$V_r = 0; V_z = 0; p' = 0; \frac{2V_\varphi}{\omega r} = \frac{T'}{T_0}; \rho' = -\rho_0 \frac{T'}{T_0}, \quad (30)$$

with an arbitrary temperature perturbation T' . In such a wave Coriolis force due to the azimuthal velocity perturbation compensates an additional centrifugal force due to density perturbation. Pressure distribution remains unperturbed. It is easy to understand this result.

From equation (30) one can obtain

$$\frac{2V_\varphi}{\omega r} = \frac{2\omega'}{\omega} = \frac{T'}{T_0}, \quad (31)$$

$$2 \ln \omega = \ln T + \text{const}, \quad (32)$$

$$\frac{\omega^2}{T} = \text{const}. \quad (33)$$

Equation 33 means that in the entropy wave the exponent $\exp\left(\frac{M\omega^2}{2RT}(r^2 - a^2)\right)$ remains unperturbed. Pressure and density satisfy to the following equations

$$p = p_w \exp\left(\frac{M\omega^2}{2RT}(r^2 - a^2)\right), \quad \rho = \rho_w \exp\left(\frac{M\omega^2}{2RT}(r^2 - a^2)\right), \quad (34)$$

where ρ_w is the density at the rotor wall. According to (30) pressure is unperturbed. Density is perturbed due to perturbation of ρ_w like in the conventional entropy wave in accordance with equation of state for the ideal gas, see eq. (28).

So, we have totally five waves in the rotating gas: two ‘‘upper’’ family waves propagating in the opposite directions, two ‘‘lower’’ family waves also propagating in the opposite directions and the fifth wave corresponding to entropy wave. The last wave has zero propagation velocity. But unlike the wave in the quiescent gas the entropy wave perturbs the azimuthal velocity of rotation of the gas.

Only one mode of the waves from the ‘‘upper’’ family has a damping length more than a few wavelengths. This wave has conventional dispersion relation of the form $\Omega = kc$ and propagates exactly along the axis of rotation. In this work we focus our attention on the damping of this wave only.

2.2. Damping of the acoustic wave

The particular wave from upper family with dispersion law $\Omega = kc$ we call acoustic wave. Remarkable property of the wave is that the energy of the wave is concentrated near the wall. Therefore this wave has the smallest damping coefficient and can propagate on the largest distance. Damping of the waves occurs due to the viscous friction of the gas and thermal conductivity. The damping of the waves occurs due to the dissipative processes in the volume of the gas and due to interaction of the gas with the wall of the rotor. Our estimates show [11, 12] that the damping of the wave occurs predominantly due to interaction of the wave with the wall of the rotor. Decrement of damping of the wave due to this effect has a form

$$\gamma_d^{surf} = \gamma_2 A \frac{(2 - \gamma)}{\gamma} \left(1 - \exp \left(-A \frac{(2 - \gamma)}{\gamma} \right) \right), \quad (35)$$

where γ_2 is the surface damping coefficient [7]

$$\gamma_2 = \frac{\sqrt{\Omega}}{\sqrt{2}ac} \left(\sqrt{\frac{\eta}{\rho}} + (\gamma - 1) \sqrt{\frac{\kappa}{\rho c_p}} \right). \quad (36)$$

and

$$A = \frac{M\omega^2 a^2}{2RT_0}. \quad (37)$$

Here η is dynamical viscosity of the gas, κ is the thermal conductivity. Estimates in [12] show that at the typical parameters the wave can propagate from one end cup of the rotor to another end cup of the rotor. The length of propagation of the wave grows with the pressure as \sqrt{p} , while the optimal pressure is proportional to length of the rotor L . Inevitably at some length of the rotor the wave will not reach opposite end of the rotor. According to our estimates this happens at the length of the rotor $L = 0.8$ m at the parameters of the IGUASU GC.

3. Impact of the waves on the dynamics of the gas

At present time our numerical modeling of dynamics of the gas in GC in presence of the waves show that the acoustic wave affects on the gas content, axial circulation of the gas and can result into resonance phenomena.

3.1. Gas content

The gas content in the GC is determined basically by the feed flow, by the geometry of the baffle separating the working chamber from the product chamber and partially by the pressure in the product chamber. Perturbation of the pressure and density of the gas by the wave leads to more efficient flow from the working chamber to the product chamber. Our calculations show that the wave can provide 15 % increase of the product flow. Correspondingly, this will result into 15% decrease of the gas content in the GC [13].

3.2. Axial circulation

It is well known that the waves can produce the flow of gas due to their absorption [14]. These are so-called acoustic flows described by Lord Rayleigh [15]. They are produced due to transfer of the energy and momentum from the waves to the gas due to the molecular viscosity. This way the waves generated by the scoops can provide an additional mechanism of generation of the secondary axial circulation in GC. This mechanism can essentially differ from the conventional mechanism of the circulation generation. The waves can propagate and therefore transfer the breaking torque at larger distance from the scoops than it happens at the axisymmetric brake.

Exploration of this mechanism can change efficiency of the isotope separation and working parameter of the GC.

The influence of the linear waves on the hydrodynamic flow in the GC was investigated in [16]. We obtained that the pulsating breaking force almost twice increases the mass flux in the axial circulation flow in compare with the mass flux at the stationary breaking force in the closed GC with two cameras. Conventional axisymmetric models of the gas flow in the GC do not take into account this effect. This could result into incorrect estimate of the optimal breaking force in the axisymmetric model of sinks and sources [17].

3.3. Resonances

The scoops in the GC produce waves propagating towards each other. Their interference can produce effects of resonances when energy density of the wave increases at some length of the rotor. The resonance occurs at the length of the rotor L satisfying to the condition $L = \frac{\lambda}{2n}$, where λ is the wave length and n is an integer number. In work [18] the resonances were discovered. The resonances repeat at the rotor length variation equal to the half of wavelength. The impact of the resonances on the separative efficiency of GC would be the most interesting and important step in investigation of the gas dynamics in strong centrifugal fields.

4. Impact of the waves on the separation process

Our experience tells us that frequently vibration promotes separation. Therefore, the idea to use waves in the gas for separation of isotopes looks rather attractive and reasonable. This idea is supported by experiments [19, 20] and theoretical analysis performed by specialists from Los Alamos [21]. They discovered rather interesting phenomena. Sound waves propagating through thin tubes perform separation of mixture of gases and in particular mixture of isotopes. Their theoretical analysis shows that this phenomena occurs due to interaction of the sound wave with the wall of the tube. Difference in the temperature between the wall and gas in the wave result into thermal diffusion of the mixture of the gases and finally to the transport of one component into direction of propagation of the wave and other component into opposite direction.

The conditions in the rotating gas promotes this effect because the gas is compressed to the wall and there is a strong gradient of pressure along radius. Therefore the effect discovered in Los Alamos could be essentially amplified in the gas centrifuge. It would be extremely tempting to use this effect to amplify the efficiency of the gas centrifuges.

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